PREFACE

In the ACs' conference held in July, 2010 at KVS (HQ), New Delhi, issue of Study Material for Board classes was discussed at length and finally decided to provide it to students. Various Regional Offices were asked to prepare the study material in different subjects while the task of its correction and moderation was assigned to various ZIETs of KVS.

KVS, ZIET, Chandigarh received study material in the subjects of Physics, Chemistry, and Biology &Maths for XII, Maths and Science &Tech. for X class, from various Regional Offices. The study material was got reviewed and suitably modified by organising workshops of experienced and competent subject teachers with the co-operation and guidance of AC,KVS,RO,CHD. Corrected study material was sent to all regional offices for providing it to students and also uploaded on the Website WWW.zietchandigarh.org.

Subject teachers, both at the preparation and moderation levels have done a remarkable job by preparing a comprehensive study material of multiple utility .It has been carefully designed and prepared so as to promote better learning and encourage creativity in students through their increased self efforts for solving assignments of different difficulty level. But the teachers and the students must bear in mind that the purpose of the study material is in no way to replace the text-book, but to make it a complete set by supplementing it with this study material so that it may provide requisite and adequate material for use in different ways.

The study material can be effectively used in the following ways:

- **Practice material** to supplement questions given in the textbook.

- **Material for Study Camps:** The purpose of conducting study camps is to inculcate study habits amongst students under active supervision of the teachers. These camps can beorganised within the normal school hours and days. Day wise target will be ascertained and given to the students and reviewed by the concerned subject teacher. If the target is not achieved by any student, it will be added to the next day's target.

- **Master Cards:** The teachers can help students prepare master cards by taking the important questions/topics/points/concepts /reactions/terms etc from this study material for the quick revision for the examination.
- **Crash Revision Courses:** The material can also be used for preparing handouts for conducting Crash Revision Courses under the supervised guidance of the teachers just before or in the gaps between papers during examination.

Effectiveness of the study material will ultimately depend upon its regular and judicious use for the above listed purposes both by teachers and students. While attempting the source material, it would be quite useful to mark every time a question done successfully with a tick out (✓) and a question not done successfully with a dot (•). It can be later used as a source of feedback for error analysis and for effective subsequent revisions/remedial work etc. I am sure that this well prepared study material if used sincerely and judiciously will surely bring cheers to all sections of students.

I, also, take this opportunity to extend my most sincere gratitude to our Hon'ble, Commissioner KVS (HQ), New Delhi, and other higher authorities of KVS for providing this opportunity for making some useful contribution to the study material.

I also extend my thanks to all the Assistant Commissioners of various Regions for their in-valuable contribution in preparation of the Study Material in various subjects.

Above all, sincere and dedicated efforts of the subject teachers in preparation of this study material deserve full appreciation. Teacher's observations, suggestions and critical analysis for further improvement of the study material mailed to 'kvszietchd' @gmail.com, will be highly appreciated.

With best wishes to all users of this STUDY MATERIAL.

(HAR GOPAL)
Director

KVS ZIET Chd.

STUDY MATERIAL

SUBJECT: MATHEMATICS

CLASS :XII

KENDRIYA VIDYALAYA SANGATHAN

REGIONAL OFFICE CHANDIGARH

YEAR 2010-2011

INDEX

Topic wise Analysis of Examples and Questions
NCERT TEXT BOOK

Chapters	Concepts	Number of Questions for revision		Total
		Questions From Solved Examples	Questions From Exercise	
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02	**Inverse Trigonometric Functions**	**05**	**09**	**14**
03	**Matrices & Determinants**	**06**	**25**	**31**
04	**Continuity& Differentiability**	**08**	**21**	**29**
05	**Application of Derivative**	**06**	**14**	**20**
06	**Indefinite Integrals**	**17**	**45**	**62**
07	**Applications of Integration**	**05**	**09**	**14**
08	**Differential Equations**	**07**	**19**	**26**
09	**Vector Algebra**	**07**	**18**	**25**
10	**Three Dimensional Geometry**	**07**	**12**	**19**
11	**Linear Programming**	**09**	**12**	**21**
12	**Probability**	**19**	**27**	**46**
	TOTAL	**111**	**236**	**347**

Detail of the concepts to be mastered by every child of class XII with exercises and examples of NCERT Text Book.

SYMBOLS USED

*** : Important Questions, ** :Very Important Questions, *** : Very-Very Important Questions**

S.No	Topic	Concepts	Degree of importance	Refrences
1	**Relations & Functions**	1.Domain , Co-domain & Range of a relation	*	NCERT Text Book XII Ed. 2007(Previous Knowledge)
		2.Types of relations	***	NCERT Text Book XII Ed. 2007 Ex 1.1 Q.No- 5,9,12
		3.One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
		4.Composition of function	*	Ex 1.3 QNo- 7,9,13
		5.Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11
2	**Inverse Trigonometric Functions**	1.Principal value branch Table	**	NCERT Text Book XII Ed. 2007 Ex 2.1 QNo- 11, 14
		2. Properties of Inverse Trigo-nometric Functions	***	Ex 2.2 QNo- 7,13, 15 Misc Ex Q.No.9,10,11,12
3	**Matrices & Determinants**	Order, Addition, Multiplication and transpose of matrices	***	NCERT Text Book XII Ed. 2007 Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
		Cofactors & Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
		Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4–Q.No 4,5,8,12,15
		To find difference between $\|A\|$, $\|adj\ A\|$, $\|kA\|$, $\|A.adjA\|$	*	Ex 4.1 –Q.No 3,4,7,8
		Properties of Determinants	**	Ex 4.2–Q.No 11,12,13 Example –16,18
4	**Continuity& Differentiability**	1.Limit of a function	*	NCERT Text Book XI Ed. 2007
		2.Continuity	***	NCERT Text Book XII Ed. 2007 Ex 5.1 Q.No- 21, 26,30
		3.Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
		4.Logrithmic Differentiation	***	Ex 5.5 QNo- 6,9,10,15
		5 Parametric Differentiation	***	Ex 5.6 QNo- 7,8,10,11
		6. Second order derivatives	***	Ex 5.7 QNo- 14,16,17
		7. M. V.Thm	**	Ex 5.8 QNo- 3,4

5	**Application of Derivative.**	1.Rate of change	*	NCERT Text Book XII Ed. 2007Example 5 Ex 6.1 Q.No- 9,11
		2.Increasing & decreasing functions	***	Ex 6.2 Q.No- 6 Example 12,13
		3.Tangents & normal	**	Ex 6.3 .NET- 5,8,13,15,23
		4.Approximations	*	Ex 6.4 No- 1,3
		5 Maxima & Minima	***	Ex 6.5Q.No- 8,22,23,25 Example 35,36,37
6	**Indefinite Integrals**	(I)Integration using standard results	*	Text book of NCERT, Vol. II 2007 Edition Exp 8&9 Page 311
		(ii) Integration by substitution	*	Text book of NCERT, Vol. II 2007 Edition Exp 5&6 Page301,303
		(iii) Application of trigonometric function in integrals	**	Text book of NCERT, Vol. II 2007 Edition Exp 7 Page 306,Exercice Q13&Q24 Exercice 7.4
		(iv) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	**	Text book of NCERT, Vol. II 2007 Edition Exp. 8, 9, 10 Page 311,312 Exercice 7.4 Q 3,4,8,9,13&23
		(v)Partial Fraction	**	Text book of NCERT, Vol. II 2007 Edition Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
		(vi) Integration by Parts	**	Text book of NCERT, Vol. II 2007 Edition Exp 18,19&20 Page 325
		(Vii)Some Special Integrals $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2 - a^2}\, dx$	***	Text book of NCERT, Vol. II 2007 Edition Exp 23 &24 Page 329
		(viii)Definite Integrals based upon types of indefinite integrals	*	Text book of NCERT, Vol. II 2007 Edition Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9

	Definite Integrals	(ix) Definite integrals as a limit of sum	**	Text book of NCERT, Vol. II 2007 Edition Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8
		(x) Properties of definite Integrals	***	Text book of NCERT, Vol. II 2007 Edition Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21
		(xi) Integration of modulus function	**	Text book of NCERT, Vol. II 2007 Edition Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11
7	**Applications of Integration**	Area Under the Curve y=f(X) and the lines x=a and x=b ,then area= $\int_a^b y dx$	*	NCERT Text Book Edition 2007 Ex.8.1{Q.1,2}
		Area under the curve x=f(y) and the lines y=c, y=d then area = $\int_c^d x dy$	*	Ex.8.1 (Q.3}
		Area of the region enclosed between two curves y =f(x) and y=g(x) and the lines x-a.and x = b then Area=$\int_a^b (f(x) - g(x))dx$	**	Page364,368 eg4,7 Ex 8.1Q 10 ,Ex8.2 (Q1 ,2,3},Misc Ex Q(13 ,14}
		If f(x).$\geq$ g(x) in [a,c] andf(x)$\leq$ g(x) in [c,b] a,< c <b then area =$\int_a^c (f(x) - g(x))dx$ +$\int_c^b (f(x) - g(x))dx$	***	Page370,373,374 e.g. 9,13,14 Ex8.2 (Q4,5}Misc exQ 13,14
		Rough sketch of the shaded portion of the curve	*	
		Points of the Intersection of curves	*	
		Area of the regions bounded by		
		(1) Curve (Parabola, Ellipse, Circle) and a line	*	e.g.3,4,14 ,Misc Ex (Q1,2,3,6,7,8,9,10}
		Two Circles	**	e.g.10,Ex8.2 Q2
		Circle and Parabola	**	e.g.7,Ex8.2Q1, Misc Ex Q15
		Vertices of a Triangle	***	e.g.9 ,Ex8.2 Q1,Misc Ex Q13
		Sides of a Triangle	***	Ex.8.2(Q5},Misc Ex Q14
		Two Parabolas	**	e.g.6,13
8.	**Differential Equations**	1Order and degree of a differential equation	***	Q. 3,5,6 pg 382
		2.General and particular solutions of a differential equation	**	Ex. 2,3 pg384
		3.Formation of differential equation whose general	*	Q. 7,8,10 pg 391

		solution is given		
		4.Solution of differential equation by the method of separation of variables	*	Q.4,6,10 pg 396
		5.Homogeneous differential equation of first order and first degree	**	Q. 3,6,12 pg 406
		Solution of differential equation of the type dy/dx +py=q where p and q are functions of x And solution of differential equation of the type dx/dy+px=q where p and q are functions of y	***	Q.4,5,10,14 pg 413,414
9.	**Vector Algebra**	Vector and scalars	*	Q2 pg 428
		Direction ratio and direction cosines	*	Q 12,13 pg 440
		Unit vector	* *	Ex 6,8 Pg 436
		Position vector of a point and collinear vectors	* *	Q 15 Pg 440 Q 11 Pg440 Q 16 Pg448
		Dot product of two vectors	**	Q6 ,13 Pg445
		Projection of a vector	* * *	Ex 16 Pg 445
		Cross product of two vectors	* *	Q 12 Pg458
		Area of a triangle	*	Q 9 Pg 454
		Area of a parallelogram	*	Q 10 Pg 455
10	**Three Dimensional Geometry**	Direction Ratios and Direction Cosines	*	Ex No 2 Pg -466 Ex No 5 Pg – 467 Ex No 14 Pg - 480
		Cartesian and Vector equation of a line in space & conversion of one into another form	**	Ex No 8 Pg -470 Q N. 6, 7, - Pg 477 QN 9 – Pg 478
		Co-planer and skew lines	*	Ex No 29 Pg -496
		Shortest distance between two lines	***	Ex No 12 Pg -476 Q N. 16, 17 - Pg 478
		Cartesian and Vector equation of a plane in space & conversion of one into another form	**	Ex No 17 Pg -482 Ex No 18 Pg – 484 Ex No 19 Pg – 485 Ex No 27 Pg – 495 Q N. 19, 20 - Pg 499
		Angle Between (i) Two lines (ii) Two planes (iii) Line & plane	 * * **	Ex No 9 Pg -472 Q N. 11 - Pg 478 Ex No 26 Pg – 494 Q N. 12 - Pg 494 Ex No 25 Pg - 492
		Distance of a point from a plane	**	Q No 18 Pg -499 Q No 14 Pg – 494
		Distance measures parallel to plane and parallel to line	**	
		Equation of a plane through	***	Q No 10 Pg -493

		the intersection of two planes		
		Foot of perpendicular and image with respect to a line and plane	**	Ex. N 16 Pg 481
11	**Linear Programming**	(i) LPP and its Mathematical Formulation (ii) Graphical method of solving LPP (bounded and unbounded solutions) (iii) Types of problems (a) Diet Problem (b) Manufacturing Problem (c) Allocation Problem (d) Transportation Problem (e) Miscellaneous Problems	** ** *** *** ** * **	**Articles 12.2 and 12.2.1** **Article 12.2.2** Solved Examples 1 to 5 Q. Nos 5 to 8 Ex.12.1 Q. Nos 1, 2 and 9 Ex. 12.2 Solved Example 9 Q. Nos 2 and 3 Misc. Ex. Solved Example 8 Q. Nos 3,4,5,6,7 of Ex. 12.2 Solved Example10 Q. Nos 4 & 10 Misc. Ex. Solved Example 7 Q. No 10 Ex.12.2, Q. No 5 & 8 Misc. Ex. Solved Example11 Q. Nos 6 & 7 Misc. Ex. Q. No 8 Ex. 12.2
12	**Probability**	(i) Conditional Probability (ii)Multiplication theorem on probability (iii) Independent Events (iv) Baye's theorem, partition of sample space and Theorem of total probability (v) Random variables & probability distribution Mean & variance of random variables (vi) Bernoulli,s trials and Binomial Distribution	*** ** *** *** *** ***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 to 15 Ex. 13.1 **Article 13.3** Solved Examples 8 & 9 Q. Nos 2, 3, 13 14 & 16 Ex.13.2 **Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2 **Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex. **Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4 **Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 to 13 Ex.13.5

TOPIC 1

RELATIONS & FUNCTIONS

SCHEMATIC DIAGRAM

S.No	Topic	Concepts	Degree of importance	References
1	Relations & Functions	1.Domain , Co domain & Range of a relation		NCERT Text Book XI Ed. 2007
		2.Types of relations	***	NCERT Text Book XII Ed. 2007 Ex 1.1 Q.No- 5,9,12
		3.One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
		4.Composition of function	*	Ex 1.3 QNo- 7,9,13
		5.Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11

Assignment on concepts 1, 2, 3, 4 & 5

LEVEL I

1. If A = {1,2,3,4,5}, write the relation a R b such that a + b = 8, a ,b € A. Write the domain, range & co-domain. Ans R = { (3,5),(4,4),(5,3) }

2. Let R be the relation in the set N given by R = {(a,b)| a = b – 2 , b > 6}
Whether the relation is reflexive or not ? justify your answer.

3.Show that the relation R in the set N given by
R = {(a , b)| a is divisible by b , a , b $\in$ N} Is reflexive and transitive but not symmetric.

4. Let R be the relation in the set N given by R = {(a ,b)| a > b}
Show that the relation Is neither reflexive nor symmetric but transitive.

5. Let R be the relation on R defined as (a , b) ϵR iff 1+ ab > 0 $\forall$ a,b ϵR.
(a) Show that R is symmetric.
(b) Show that R is reflexive.
(c) Show that R is not transitive.

6. Check whether the relation R is reflexive, symmetric and transitive.
R = { (x ,y)| x – 3y=0} on A ={1,2,3……….13,14}.

7. Let f : R →R & g : R→R be defined as f(x) = x^2 , g(x) = 2x – 3 . Find fog(x).
Ans .$4x^2 - 12x + 9$

8. If f: R→R defined as f(x) = $\frac{2x-7}{4}$ is an invertible function . Find $f^{-1}(x)$.

Ans $f^{-1}(x) = \frac{(4x+7)}{2}$

9.Write the number of all one-one functions on the set A={a,b,c} to itself. Ans.6
10. Let * be the binary operation on N given by a*b = LCM of a & b . Find 3*5. Ans15

11. Let *be the binary on N given by a*b =HCF of a ,b , a,b $\in$N. Find 20*16. Ans.4

12. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.
Write the identity of *, if any. Ans.e = 5

13. If a binary operation '*' on the set of integer Z , is defined by $a * b = a + 3b^2$
Then find the value of 2 * 4. Ans.50

14. Show that the function f: R→R defined by $f(x)=x^2$ is neither one-one nor onto.

15. Show that the function f: N→N given by f(x)=2x is one-one but not onto.

16. Show that the signum function f: R→R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

is neither one-one nor onto.

17. Let A = {-1,0,1} and B = {0,1}. State whether the function f : A → B defined by $f(x) = x^2$ is bijective.

LEVEL 2

1. Show that the relation R on A ,A = { x| x $\in$ Z , $0 \le x \le 12$ } ,
R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

2.Let N be the set of all natural numbers & R be the relation on N × N defined by
{ (a,b) R (c,d) iff a + d = b + c}. Show that R is an equivalence relation.

3.Show that the relation R in the set A of all polygons as:
R ={(P_1,P_2), P_1 & P_2 have the same number of sides} is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?

4.Show that function f : R→R defined by $f(x)= 7-2x^3$ for all x $\in$R is bijective.

5.If f: R→R is defined by $f(x)= \frac{3x+5}{2}$. Find f^{-1}. Ans $f^{-1}(x) = \frac{(2x-5}{3}$

6. If $f(x) = e^{2x}$ and $g(x) = \log \sqrt{x}$, x > 0, find
(a) f + g(x) (b) f g(x) (c) f o g (x) (d) g o f (x).

7. If $f(x) = \frac{x-1}{x+1}$, then show that (a) $f\left(\frac{1}{x}\right) = -\ f(x)$ (b) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

8. Let f, g : R→R be defined by f(x)=|x| & g(x) = [x] where [.] denotes the greatest integer function. Find f o g (5/2) & g o f (-√2).

9. Let $f(x) = \frac{x-1}{x+1}$. Then find f(f(x)) Ans: -1/x

10. Let $f(x) = \frac{x-1}{x+1}$, x≠-1,then find $f^{-1}(x)$ Ans: $\frac{1+x}{1-x}$

11. If $y = f(x) = \frac{3x+4}{5x-3}$, then find fof(x) i.e. f(y) Ans: x

12. If $f(x) = x^2 - x^{-2}$, then find f(1/x). Ans: -f(x)

13. Let A= N×N & * be the binary operation on a A defined by (a,b) * (c,d) = (a+c, b+d)
Show that * is (a) Commutative (b) Associative (c) Find identity for * on A, if any.

14. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b).
Find: (i) the identity element of A (ii) the invertible element of A.

15.Examine which of the following is a binary operation

(i) $a * b = \frac{a+b}{2}$; a,b ∈ N (ii) $a*b = \frac{a+b}{2}$ a,b ∈ Q

for binary operation check commutative & associative.

LEVEL 3

1. Consider a function f : $R_+ \rightarrow [-5, \infty)$ defined $f(x) = 9x^2 + 6x - 5$. Show that f is invertible &
$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$, where $R_+ = (0,\infty)$.

2.Consider a function f: R→R given by f(x) = 4x + 3. Show that f is invertible & f^{-1}: R→R with $f^{-1}(y) = \frac{y-3}{4}$.

3. Show that f: R→R defined by $f(x) = x^3+4$ is one-one, onto. Show that $f^{-1}(x) = (x-4)^{1/3}$.

4.Let A= N×N & * be a binary operation on A defined by (a , b) × (c , d) = (ac , bd)
~~∀~~ (a , b),(c , d) ϵ N×N

(i) Find (2,3) * (4,1)

(ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal

(iii) Show that * is commutative & associative on A.

5.Let A = Set of all triangles in a plane and R is defined by

$R=\{(T_1,T_2) : T_1,T_2 \in A \ \& \ T_1 \sim T_2 \}$

Show that the R is equivalence relation.

Consider the right angled Δs, T_1 with size 3,4,5;
T_2 with size 5,12,13;
T_3 with side 6,8,10;

Which of the pairs are related?

6.Let R be the relation in the set N of natural numbers & defined by

R = {(x, y); x , y ϵN, x+2y =10}.

Write the relation R and check if it is symmetric, reflexive, and transitive.

7.Define a binary operation * on the set {0,1,2,3,4,5} as

$$a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & a+b \geq 6 \end{cases}$$

Show that zero in the identity for this operation & each element of the set is invertible with 6 - a being the inverse of a.

8. Show that the function f: R→ R defined by f(x) $\frac{2x-1}{3}$ = x ∈ R is one- one & onto function.
Also find the f^{-1}.

Questions for self evaluation

1. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

2. Show that each of the relation R in the set A = {x ∈ **Z** : 0 ≤ x ≤ 12}, given by
R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

3. Show that the relation R defined in the set A of all triangles as R = {(T1, T2) : T1 is similar to T2}, is equivalence relation. Consider three right angle triangles T1 with sides 3, 4, 5, T2 with sides 5, 12, 13 and T3 with sides 6, 8, 10. Which triangles among T1, T2 and T3 are related?

4. If R1 and R2 are equivalence relations in a set A, show that R1 $\cap$ R2 is also an equivalence relation.

5. Let A = **R** – {3} and B = **R** – {1}. Consider the function f : A $\rightarrow$ B defined by f (x) = $\left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

6. Consider f : **R+** $\rightarrow$ [– 5, ∞) given by f (x) = $9x^2 + 6x - 5$. Show that f is invertible and find f^{-1}.

7. On R – {1} a binary operation '$*$' is defined as $a * b = a + b - ab$. Prove that '$*$' is commutative and associative. Find the identity element for '$*$'.Also prove that every element of R – {1} is invertible.

8. If A = $Q \times Q$ and '$*$' be a binary operation defined by $(a, b) * (c, d) = (ac, b + ad)$, for (a, b), (c, d)$\in$ A .Then with respect to '$*$' on A
(i) examine whether '$*$' is commutative & associative
(i) find the identity element in A,
(ii) find the invertible elements of A.

TOPIC 2
INVERSE TRIGONOMETRIC FUNCTIONS

<u>SCHEMATIC DIAGRAM</u>

2	**Inverse Trigonometric Functions**	1.Principal value branch Table	**	NCERT Text Book XII Ed. 2007 Ex 2.1 QNo- 11, 14
		2. Properties of Inverse Trigo-nometric Functions	***	Ex 2.2 QNo- 7,13, 15 Misc Ex Q.No.—9,10,11,12

Assignments on concept 1&2 :

LEVEL 1

1. Find the principal value of the following :

(i) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ Ans. $\frac{\pi}{6}$ (ii) $\sin^{-1}\left(-\frac{1}{2}\right)$ Ans. $-\frac{\pi}{6}$

(iii) $\tan^{-1}\left(-\sqrt{3}\right)$ Ans. $-\frac{\pi}{3}$ (iv) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ Ans. $\frac{3\pi}{4}$

2. Evaluate the following :

(i) $\cot[\tan^{-1} a + \cot^{-1} a$ Ans. 0 (ii) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ Ans. $\frac{2\pi}{3}$

(iii) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ Ans. -1 (iv) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ Ans. $\frac{5\pi}{6}$

3. Prove thé following :

(i) $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$

(ii) $3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right)$

(iii) $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

LEVEL II

1.Find x if

(i) $\sin\left[\sin^{-1}\frac{1}{5} + \cos^{-1}x\right] = 1$ Ans. $\frac{1}{5}$

(ii) $\sec^{-1}\left(\sqrt{2}\right) + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$ Ans. $\sqrt{2}$

2. Write the following in simplest form :

(i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$ Ans. $\frac{1}{2}\tan^{-1}x$

(ii) $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$, $|x| < a$ Ans. $\sin^{-1}\frac{x}{a}$

(iii) $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$ Ans. $\frac{x+y}{1-xy}$

(v) $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$ Ans. x

LEVEL III

1. Prove the following :

(i) $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

(ii) $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

(iii) $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

(iv) $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

2. Solve the following :

(i) $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$ Ans. $\pm\frac{1}{\sqrt{2}}$

(ii) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$ Ans. $\frac{\pi}{4}$

(iii) $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$ Ans. $\frac{1}{\sqrt{3}}$

(iv) $\sin^{-1}(1-x)-2\sin^{-1}x=\pi/2$ Ans. 0

(v) $\tan^{-1}2x+\tan^{-1}3x=\pi/4$ Ans. $1/6$

(vi) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$ Ans $1/4$

Questions for self evaluation

1. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\quad x\in\left[-\frac{1}{\sqrt{2}},1\right]$

3. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

4. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

5. Prove that $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$

6. Write in the simplest form $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

7. Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

8. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

TOPIC 3
MATRICES & DETERMINANTS

SCHEMATIC DIAGRAM

S.No	Topic	Concepts	Degree of importance	References
3	Matrices & Determinants	Order, Addition ,Multiplication and transpose of matrices	***	NCERT Text Book XII Ed. 2007 Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
		Cofactors & Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
		Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4–Q.No 4,5,8,12,15
		Basic difference between $\lvert\ \rvert$ & [ ], $\lvert \text{adj } A \rvert$, $\lvert kA \rvert$, $\lvert A \text{ adj} A \rvert$	*	Ex 4.1 –Q.No 3,4,7,8
		Properties of determinents	**	Ex 4.2–Q.No 11,12,13 Example –16,18

Level 1

1. A matrix A of order 3 × 3 has determinant 5. What is the value of $\lvert 3A \rvert$. Ans.135
2. For what value of x the matrix is singular $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ Ans. 3
3. If A is a square matrix of order 3 such that $\lvert \text{adj } A \rvert = 64$, Find $\lvert A \rvert$. Ans. ±8
4. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the value of $\lvert A^2 - 2A \rvert$. Ans. 25
5. If $\begin{vmatrix} 3 & m \\ 4 & m \end{vmatrix} = 3$, find the value of m. Ans. 3
6. Give example of matrices A & B such that AB = O, but BA ≠ O

Level 2

1. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = 0$. Thus find A^{-1}.

Ans. $\frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

2. If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

Ans. x = 8, y = 8 and $A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

Using properties of determinants, prove the following :

3. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

4. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

5. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

10 $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

12. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

LEVEL 3

1. Solve the following system of equations by matrix method, where $x \neq 0, y \neq 0, z \neq 0$:

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10, \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13.$$

2. Find A^{-1}, where A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Hence solve the equations

x + y + 2z = 0, x + 2y – z = 9 and x – 3y + 3z = –14 .

3. Find the product AB, where A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the equations x – y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1

4. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

5. Solve the following system of equations by matrix method :

x + 2y + z = 7, x + 3z = 11, 2x – 3y = 1

Using properties of determinants, prove the following :

6. Prove That $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

7. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ [Hint: Apply $C_1 \to -bC_3$ and $C_2 \to aC_3$]

8. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$ [Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively]

9. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

[Hint : Apply $R_1 \to R_1 + R_3$ and take common a + b + c]

10. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$ [Hint : Apply $R_1 \to aR_1$, $R_2 \to bR_2$, and $R_3 \to cR_3$]

11. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

[Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively and then apply $R_1 \to R_1 + R_2 + R_3$]

12. If p, q, r are not in G.P and $\begin{vmatrix} 1 & \frac{q}{p} & \alpha+\frac{q}{p} \\ 1 & \frac{r}{q} & \alpha+\frac{r}{q} \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, show that $p\alpha^2+2p\alpha+r=0$.

13. If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ Show that either a + b +c = 0 or a = b = c.

Questions for self evaluation

1. Using properties of determinants, prove that : $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

2. Using properties of determinants, prove that : $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

3. Using properties of determinants, prove that : $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

4. . Express $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

5. Let A = $\begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$, prove by mathematical induction that : $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$.

6. If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

7. Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

8. Solve the following system of equations : x + 2y + z = 7, x + 3z = 11, 2x – 3y = 1.

9. Find the product AB, where A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the equations x – y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1.

10. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

TOPIC 4
CONTINUITY& DIFFERENTIABILITY
SCHEMATIC DIAGRAM

S.No	Topic	Concepts	Degree of importance	References
4	Continuity& Differentiability	1.Limit of a function		NCERT Text Book XI Ed. 2007
		2.Continuity	***	NCERT Text Book XII Ed. 2007 Ex 5.1 Q.No- 21, 26,30
		3.Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
		4.Logrithmic Differentiation	***	Ex 5.5 QNo- 6,9,10,15
		5 Parametric Differentiation	***	Ex 5.6 QNo- 7,8,10,11
		6. Second order derivatives	***	Ex 5.7 QNo- 14,16,17
		7. Mean Value Theorem	**	Ex 5.8 QNo- 3,4

LEVEL-1

1 MARK QUESTIONS

1.Examine the continuity of the function $f(x)=x^2+5$ at $x=-1$. Ans. Continuous

2. Examine the continuity of the function $f(x)=1/(x+3), x\in R$. Ans. Not Continuous

3. Give an example of a function which is continuous at x=1, but not differential at x=1.

4. Show that f(x) = 4x is a continuous for all $x\in R$.

5. For what value of k,the function $\begin{cases} kx^2, if & x\le 2 \\ 3, if & x>2 \end{cases}$, is continuous at x = 2 . Ans.3/4

4 MARK QUESTIONS

1. Show that the function $f(x)=\begin{cases} X^3+3, & x\ne 0 \\ 1, & x=0 \end{cases}$ is not continuous at x=0.

2. Is the function 'f' defined by $f(x)=\begin{cases} x, x\le 1 \\ 5, x>1 \end{cases}$ Continuous at x = 0? at x =1?at x = 2?

3. Discuss the continuity of the function $f(x)=\frac{1}{x}$, $(x\ne 0)$.

LEVEL-2

1. Find the value of m, for which the function $f(x)=\begin{cases} m(x^2-x) & x>0 \\ \cos x & x\le 0 \end{cases}$ is continuous at x=0.

2. For what value of k, the function f(x)= $\begin{cases} kx^2 & ,if \quad x \le 2 \\ 3 & ,if \quad x > 2 \end{cases}$ is continuous at x=2?

3. If Function f(x)= $\dfrac{2x+3\sin x}{3x+2\sin x}$, for $x \neq 0$ is continuous at x=0, then Find f(0). Ans.1

4. If f(x)= $\begin{cases} \dfrac{\sin 3x}{x}, & when \ x \neq 0 \\ 1, & when \ x = 0 \end{cases}$ Find whether f(x) is continuous at x = 0.

5. If f(x)= $\begin{cases} 3x+5, & if \quad x \ge 2 \\ 5x^2-9, & if \quad x > 2 \end{cases}$, , show that f(x) is continuous at x=2.

6. Show that function f(x)= $\begin{cases} |x|, & x \le 2 \\ |x|, & x > 2 \end{cases}$ is continuous on [0,2].

7. Examine the continuity of the function f(x)= $\begin{cases} x-|x|, & x \neq 0 \\ 2 \quad , & x = 0 \end{cases}$, at x=0. Ans. Not Continuous at x=0.

8. Show that function f defined by f(x)= $\begin{cases} 2x\,, & x < 2 \\ 2 \quad , & x = 2 \\ x^2\,, & x > 2 \end{cases}$ is discontinuous at x = 2.

LEVEL III

1.Find the value of m, for which the function f(x)= $\begin{cases} m(x^2-x) & x > 0 \\ \cos x & x \le 0 \end{cases}$ is continuous at x=0.

2.For what value of k, the function f(x)= $\begin{cases} \dfrac{1-\cos 4x}{8x^2} & , \quad x \neq 0 \\ k & , \quad x = 0 \end{cases}$ is continuous at x = 0?

Ans.1

3. Examine the continuity of the Function f(x)= $\begin{cases} \dfrac{x}{2|x|} & , \quad x \neq 0 \\ \dfrac{1}{2} & , \quad x = 0 \end{cases}$ at x = 0.

4.Let f(x)= $\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & , \quad if \quad x < \dfrac{\pi}{2} \\ a & , \quad if \quad x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & , \quad if \quad x > \dfrac{\pi}{2} \end{cases}$ If f(x) be a continuous function at x= $\dfrac{\pi}{2}$,

find a and b. Ans. a = 1/2, b = 4

5.Determine the constants a and b such that the function

$$f(x)=\begin{cases} ax^2+b, & if \quad x>2 \\ 2, & if \quad x=2 \\ 2ax-b, & if \quad x<2 \end{cases}$$, is continuous. Ans. a= ½, b = 0

6.For what value of k, is the following function continuous at x =2?

$$f(x)=\begin{cases} 2x+1 & ; x<2 \\ k & ; x=2 \\ 3x-1 & ; x>2 \end{cases}$$

Ans. K = 5

7.For what value of k, is the function f(x) continuous at x = 0?

$$f(x)=\begin{cases} \dfrac{\sin x + x\cos x}{x}, & when \quad x \neq 0 \\ k, & when \quad x=0. \end{cases}$$

Ans. K = 2

DIFFERENTIABILITY

LEVEL-1

1.Differentiate $y=\log_7(\log x)$. $y' = 1/(x\log x \log 7)$

2. If $y = 500e^{7x} + 600e^{-7x}$, Show that $\frac{d^2y}{dx^2} = 49y$.

3. Discuss the differentiability of the function $f(x)=(x - 1)^{2/3}$ at x = 1.

4. Differentiate , sin(log x),with respect to x. Ans. $\frac{\cos(\log x)}{x}$

5.Differentiate $y = x^{\tan x} + (\sin x)^{\cos x}$.

6.Differentiate $y = \tan^{-1}\frac{2x}{1-x^2}$.

7. If $y=\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$, Find $\frac{dy}{dx}$.

8. If $y.\sqrt{x^2+1} = \log[\sqrt{x^2+1} - x]$, show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$.

9. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$,show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$.

LEVEL-II

1. Differentiate, $\log_a(\sin x)$ with respect to x. Ans = tanx

2. Find $\frac{dy}{dx}$, $y = \log(\cos x^2)$. Ans = $-2x\tan x^2$

3. Find $\frac{dy}{dx}$, $y = \cos(\log x)^2$. Ans = $2\log x\sin(\log x)^2/x$

4. Verify Rolle's theorem for the function f(x) = sinx, in $[0, \pi]$.Find c, if verified.

5. Find $\frac{dy}{dx}$ of $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$.

6.Find $\frac{dy}{dx}$ of $y = \tan^{-1}\left(\frac{\sin x}{1-\cos x}\right)$.

7.Find $\frac{dy}{dx}$ of $y=\sqrt{\frac{\sec x-1}{\sec x+1}}$.

8.If $y=e^{ax}\sin bx$, then prove that $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$.

9.Find $\frac{d^2y}{dx^2}$, if $y=\frac{3at}{1+t}$, $x=\frac{2at^2}{1+t}$.

LEVEL-III

1 MARK QUESTIONS

1.Differentiate , $\cos^{-1}\sqrt{x}$,with respect to x.

2.Find the second derivative of, log x, with respect to x.

3.Given f(0)=-2,f'(0)=3.Find h'(0) where h(x)=xf(x).

4.If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$

4 MARK QUESTIONS

1.Find $\frac{dy}{dx}$ $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$.

2.Find $\frac{dy}{dx}$ $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$. $0<x<\frac{\pi}{2}$.

3.It is given that for the function $f(x) = x^3 - 6x^2 + px + q$ on[1,3] , Rolle's theorem holds with $c = 2+\frac{1}{\sqrt{3}}$. Find the values p and q.

4. If$(x-a^2)+(y-b^2)=c^2$for some c>0.Prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant, independent of a and b.

5. If $y=\sin^{-1}\left(\frac{a+b\cos x}{b+a\cos x}\right)$, Show that $\frac{dy}{dx}=\frac{-\sqrt{b^2-a^2}}{b+a\cos x}$.

6. Prove that $\frac{d}{dx}\left[\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x}{1-x^2}\right)\right]=\frac{1}{1+x^4}$.

Questions for self evaluation

1.For what value of k is the following function continuous at x = 2 ?

$$f(x)=\begin{cases}2x+1\,;\ x<2\\ k\,;\ x=2\\ 3x-1\,;\ x>2\end{cases}$$

2.If f(x) = $\begin{cases}3ax+b, & \text{if } x>1\\ 11 & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$, continuous at x = 1, find the values of a and b.

3. Discuss the continuity of $f(x) = |x-1|+|x-2|$ at x = 1 & x = 2.

4. If f(x), defined by the following is continuous at x = 0, find the values of a, b, c

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x} & , x<0\\ c & , x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & , x>0\end{cases}$$

5.If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$.

6. $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$.

7. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.

8 If $y=\sqrt{x^2+1}-\log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

9. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

10. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$

11. If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

12.If $x^p.y^q=(x+y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

TOPIC 5
APPLICATIONS OF DERIVATIVES
SCHEMATIC DIAGRAM

S.No	Topic	Concepts	Degree of importance	References
5	**Application of Derivative.**	1.Rate of change	*	NCERT Text Book XII Ed. 2007Example 5 Ex 6.1 Q.No- 9,11
		2.Increasing & decreasing functions	***	Ex 6.2 Q.No- 6 Example 12,13
		3.Tangents & normals	**	Ex 6.3 Q.No- 5,8,13,15,23
		4.Approximations	*	Ex 6.4 QNo- 1 Part (iii)
		5 Maxima & Minima	***	Ex 6.5Q.No- 8,22,23,25 Example 35,36,37,

LEVEL I

1. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When x =10cm and y = 6cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

Ans. (i) -2 cm/min. (ii) 2 cm^2/min.

2. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm. **Ans**. $\frac{1}{\pi}$ cm/s.

3. The total revenue in Rupees received from the sale of x units of a product is given by

$R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when x = 7.

Ans. Rs 208

4. A man of height 2 metres walks at a uniform speed of 5 km/h away from a lamp post which is 6 metres high. Find the rate at which the length of his shadow increases.

Ans. $\frac{5}{2}$ km / h

5. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

Ans. $b\sqrt{3}cm^2/s$

6. Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.

Ans. (3 , 2)

7. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$

LEVEL II

1. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Ans. $\frac{1}{48\pi}$ cm/s

2. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}$ (0.5). Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4. **Ans**. $\frac{35}{88}$ m / h

3. A car starts from a point P at time t = 0 seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$. Find the time taken by it to reach Q and also find distance between P and Q. **Ans**. $\frac{32}{3}$ m

4. Find the intervals in which the following functions are strictly increasing or decreasing:

(i) $f(x) = 4x^3 - 6x^2 - 72x + 30$ **Ans**. $\uparrow$ in $(-\infty,-2)\cup(3,\infty)$ and $\downarrow$ in $(-2, 3)$

(ii) $f(x) = -2x^3 - 9x^2 - 12x + 1$ **Ans**. $\uparrow$ in $(-2, -1)$ and $\downarrow$ in $(-\infty,-2)\cup(-1,\infty)$

(iii) $f(x) = (x + 1)^3 (x - 3)^3$ **Ans**. $\uparrow$ in $(1, 3)\cup(3, \infty)$ and $\downarrow$ in $(-\infty,-1)\cup(-1, 1)$

5. Show that $y = \log(1 + x) - \frac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain.

6. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$ **Ans**. y – 2x + 2 = 0, y – 2x + 2 = 0

7. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

Ans. (i) (0, 5) and (0, – 5) (ii) (2, 0) and (–2, 0).

8. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1, 1). **Ans**. y – x = 0

9. Find the equation of tangent to the curve given by $x = a\sin^3 t$, $y = b\cos^3 t$ at a point where $t=\frac{\pi}{2}$

Ans. y = 0

10. Using differentials, find the approximate value of each of the following up to 3 places of decimal : (i) $(0.009)^{\frac{1}{3}}$ **Ans**: 0.208 (ii) $(26)^{\frac{1}{3}}$ **Ans**: 2.962

LEVEL III

1. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the maximum and minimum values.

2. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible. **Ans**: 3 cm.

3. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

4. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area?

Ans: radius = $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm and height $2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

5. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

6. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

7. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

8. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank? **Ans**: Rs 1000.

9. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

10. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening. **Ans**: Length = $\frac{20}{\pi+4}$ m , bredth $\frac{10}{\pi+4}$ m.

11. A point on the hypotenuse of a triangle is at distance *a* and *b* from the sides of the triangle. Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

12. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

13. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3\tan^2\alpha$.

14. A given quantity of metal is recast into a half cylinder (i.e. with rectangular base and semi-circular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : \pi + 2$.

15. If the sum of hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.

16. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

17. The cost of fuel for running a bus is proportional to the square of the speed generated in Km/hr. It costs Rs.48 per hour when the bus is moving with a speed of 20 km/hr. What is the most economical speed if the fixed charges are Rs.108 for one hour, over and above the running charges.

18. A window is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

Questions for self evaluation

1.Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

2. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

3. Find the intervals in which the following function is strictly increasing or decreasing:
$f(x) = -2x^3 - 9x^2 - 12x + 1$

4. Find the intervals in which the following function is strictly increasing or decreasing:
$f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$

5. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is
(a) parallel to the line $2x - y + 9 = 0$ (b) perpendicular to the line $5y - 15x = 13$.

7. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

8. Using differentials, find the approximate value of each of the following up to 3places of decimal :

(i) $(26)^{\frac{1}{3}}$ (ii) $(32.15)^{\frac{1}{5}}$

9. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

10. An open topped box is to be constructed by removing equal squares from each corner of a 3 meter by 8 meter rectangular sheet of aluminum and folding up the sides. Find the volume of the largest such box.

TOPIC 6
INDEFINITE & DEFINITE INTEGRALS
SCHEMATIC DIAGRAM

S.No	Topics	Concept	Degree of Importance	References
6	**Indefinite Integrals**	(i)Integration using standard results	*	Text book of NCERT, Vol. II 2007 Edition Exp 8&9 Page 311
		(ii) Integration by substitution	*	Text book of NCERT, Vol. II 2007 Edition Exp 5&6 Page301,303
		(iii) Application of trigonometric function in integrals	**	Text book of NCERT, Vol. II 2007 Edition Exp 7 Page 306,Exercise Q13&Q24 Exercise 7.4
		(iv) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	**	Text book of NCERT, Vol. II 2007 Edition Exp 8, 9, 10 Page 311,312 Exercise 7.4 Q 3,4,8,9,13&23
		(v)Partial Fraction	**	Text book of NCERT, Vol. II 2007 Edition Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
		(vi) Integration by Parts	**	Text book of NCERT, Vol. II 2007 Edition Exp 18,19&20 Page 325
		(Vii)Some Special Integrals $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2 - a^2}\, dx$	***	Text book of NCERT, Vol. II 2007 Edition Exp 23 &24 Page 329
		(viii)Definite Integrals based upon types of indefinite integrals	*	Text book of NCERT, Vol. II 2007 Edition Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9
	Definite Integrals	(ix) Definite integrals as a limit of sum	**	Text book of NCERT, Vol. II 2007 Edition

				Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8
		(x) Properties of definite Integrals	***	Text book of NCERT, Vol. II 2007 Edition Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21
		(xi) Integration of modulus function	**	Text book of NCERT, Vol. II 2007 Edition Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11

Assigments:

Concept (i),

(a) $\int \frac{dx}{x^2+9}$ Ans. $\frac{1}{3} tan^{-1} \frac{x}{3}$ + c

(b) $\int \frac{dx}{\sqrt{x^2-a^2}}$ Ans log(x+$\sqrt{x^2-a^2}$)+ c

Concept (ii)

(a) $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ Ans. $-\cos(\tan^{-1} x)+c$

(b) $\int sin^{-1}(\frac{2x}{1+x^2})dx$ Ans. 2xtan^{-1}x – log(1+ x^2) + c

(c) $\int e^{\sqrt{x}} dx$ Ans. $2(\sqrt{x}-1)\ e^{\sqrt{x}}+\ c$

(d) $\int sin^{-1}\sqrt{\frac{x}{a+x}} dx$ Ans. (a+x) tan$^{-1}\sqrt{\frac{x}{a}}$ - $\sqrt{ax}+\ c$

(e) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$ Ans. 2 sin√x+c

(f) $\int x\sqrt{1+2x^2}$ dx Ans. $\frac{1}{6}(1+2x^2)^{3/2}+c$

(g) $\int \frac{dx}{x(\log x)^m}$ Ans. $\frac{(\log x)^{1-m}}{1-m}+c$

(h) $\int \frac{10x^9+10^x \log 10}{x^{10}+10^x} dx$ Ans. log(x^{10}+10^x) + c

(i) $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ Ans. $-2\sin(\cos^{-1}\sqrt{x})+\cos^{-1}\sqrt{x}+\frac{1}{2}\sin(2\cos^{-1}\sqrt{x})+c$

(j) $\int \frac{x^3 \sin^{-1}(x^4)}{\sqrt{1-x^4}} dx$ Ans. $\frac{-1}{2}\sqrt{1-x^4}\sin^{-1} x^4+\sqrt{1+x^4}+c$

(k) $\int \tan^{-1}(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}})dx$ Ans. $\frac{\pi}{4}x-\frac{1}{2}x\cos^{-1} x+\frac{1}{2}\sqrt{1-x^2}+c$

Concept (iii)

(a) $\int\sqrt{\frac{1-\cos 2x}{2}}dx$ Ans. –cosx +c

(b) $\int\frac{\sec^4 x}{\sqrt{\tan x}}dx$ Ans. $2\sqrt{\tan x}++\frac{2}{5}\tan^{5/2}x+c$

(c) $\int\frac{dx}{\sqrt{\sin^3 x\cos x}}$ Ans. $\frac{-2}{\sqrt{\tan x}}+c$

(d) $\int\sin^7 xdx$ Ans. $-\cos x+\cos^3 x-\frac{3}{5}\cos^5 x+\frac{1}{7}\cos^7 x+c$

(e) $\int\cos(2\cot^{-1}\sqrt{\frac{1-x}{1+x}})dx$ Ans. $\frac{-x^2}{2}+c$

(f) $\int\frac{\sin 2x}{(a^2+b^2\sin^2 x)^2}dx$ Ans. $\frac{-1}{b^2(a^2+b^2\sin^2 x)}+c$

(g) $\int\frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ Ans. $\tan^{-1}(\tan^2 x) + c$

(h) $\int\frac{\cos x}{\cos 3x}dx$ Ans. $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + c$

(i) $\int\frac{dx}{\sin x+\sqrt{3}\cos x}$ Ans. $\frac{1}{2}\log\left|\ \operatorname{cosec}(x+\frac{\pi}{3})-\cot(x+\frac{\pi}{3})\ \right|+c$

(j) $\int\frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ Ans. $\frac{1}{ab}\tan^{-1}(\frac{a}{b}\tan x)+c$

Concept (iv)

(a) $\int\frac{dx}{\sqrt{8+3x-x^2}}$ Ans. $\sin^{-1}(\frac{2x-3}{\sqrt{41}})+c$

(b) $\int\frac{x}{\sqrt{1-x^2+x^4}}dx$ Ans. $\frac{1}{2}\log\left|(2x^2-1)+2\sqrt{x^4-x^2+1}\ \right|+c$

(c) $\int\frac{dx}{\sqrt{5x^2-2x}}$ Ans. $\frac{1}{\sqrt{5}}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\ \right|+c$

Concept (v)

(a) $\int\frac{dx}{x(x^4-1)}$ Ans. $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\ \right|+c$

(b) $\int\frac{x}{(x+1)(x+2)}dx$ Ans. $\log(\frac{(x+2)^2}{(x+1)}+c$

(c) $\int\frac{(x^2+1)}{(x^2-5x+6)}dx$ Ans. x-5log(x-2) +10log(x-3)+c

(d) $\int\frac{1}{x^4-1}dx$ Ans. $\frac{1}{4}\log\ (\frac{(x-1)}{(x+1)}-\frac{1}{2}\tan^{-1}x+c$

Concept (vi)

(a) $\int e^{2x}\sin x dx$ Ans. $\frac{e^{2x}}{5}(2\sin x-\cos x)+c$

(b) $\int e^{x}(\frac{1}{x^2}-\frac{2}{x^3})dx$ Ans. $\frac{e^x}{x^2}+c$

(c) $\int(\frac{1+\sin x}{1+\cos x})e^x dx$ Ans. $e^x\tan\frac{x}{2}+c$

(d) $\int\frac{\log x}{(1+\log x)^2}dx$ Ans. $\frac{x}{1+\log x}+c$

(e) $\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$ Ans. $\frac{2}{\pi}[(2x-1)\sin^{-1}\sqrt{x}+\sqrt{x-x^2}]-x+c$

(f) $\int\tan^{-1}x dx$ Ans. $x\tan^{-1}x-\frac{1}{2}\log(1+x^2)+c$

(g) $\int x(\log x)^2 dx$ Ans. $\frac{x^2}{4}\{2(\log x)^2-2\log x+1\}+c$

(h) $\int\cos\sqrt{x}dx$ Ans. $2(\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x})+c$

Concept (vii)

(a) $\int(16x^2-9)^{-1/2}dx$ Ans. $\frac{1}{4}\log\left|4x+\sqrt{16x^2-9}\right|+c$

(b) $\int\sqrt{x^2+3x}dx$ Ans. $\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right|+c$

(c) $\int e^x\sqrt{e^{2x}+1}dx$ Ans. $\frac{1}{2}(e^x\sqrt{e^{2x}+1}+\log|e^x+\sqrt{e^{2x}+1}|)+c$

(d) $\int\frac{dx}{x^4+x^2+1}$ Ans. $\frac{1}{2\sqrt{3}}\tan^{-1}(\frac{x^2-1}{\sqrt{3}x})-\frac{1}{4}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|+c$

(e) $\int\frac{dx}{x^4+16}$ Ans. $\frac{1}{32\sqrt{2}}\{2tan^{-1}(\frac{x^2-4}{2\sqrt{2}x})-\log(\frac{x^2-2\sqrt{2}x+4}{x^2+2\sqrt{2}x+4})\}+c$

Concept (viii)

(a) $\int_1^{\sqrt{3}}\frac{dx}{x^2+1}$ Ans. $\frac{\pi}{12}$

(b) $\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}$ Ans. $\frac{4\sqrt{2}}{3}$

(c) $\int_1^3\frac{dx}{x^2(x+1)}$ Ans. $\frac{2}{3}+\log(\frac{2}{3})$

(d) $If\int_0^k\frac{dx}{2+8x^2}dx=\frac{\pi}{16}$ find k Ans. $\frac{1}{2}$

(e) $\int_{\pi/2}^{\pi}e^x(\frac{1-sinx}{1-cosx})dx$ Ans. $e^{\pi/2}$

Concept (ix)

(a) $\int_0^5(x+1)dx$ Ans. $\frac{35}{2}$

(b) $\int_1^4(x^2-x)dx$ Ans. $\frac{27}{2}$

Concept (x)

(a) $\int_0^{\pi/2}\frac{cos^{5/2}x}{Sin^{5/2}x+cos^{5/2}x}dx$ Ans. $\frac{\pi}{4}$

(b) $\int_0^1 tan^{-1}(\frac{(2x-1)}{1+x-x^2})dx$ Ans. 0

(c) $\int_0^{\pi/2}\log(sinx)dx$ Ans. $\frac{\pi}{2}log\frac{1}{2}$

(d) $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$ Ans. $\frac{\pi^2}{4}$

Concept (xi)

(a) $\int_{-5}^{5} |x+2| dx$ Ans. 29

(b) $\int_{-1}^{3/2} |x\sin(\pi x)| dx$ Ans. $\frac{3}{\pi}+\frac{1}{\pi^2}$

(c) $\int_1^4 |x-1|+|x-2|+|x-3| dx$ Ans. $\frac{19}{2}$

(d) $\int_2^8 |x-5| dx$ Ans.9

Some Questions Other Than Text books

(a) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ Ans. $2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left|\sqrt[6]{x}+1\right|$ +c

(b) $\int \frac{dx}{x^{1/2}-x^{1/4}}$ Ans. $2\sqrt{x}+4x^{1/4}+4\log\left|x^{1/4}-1\right|$ + c

(c) $\int 9^{\log_3 x}$ dx Ans. $\frac{x^3}{3}+c$

(d) $\int e^{3\log x}(x^2)$ dx Ans. $\frac{x^6}{6}+c$

(e) $\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$ Ans. $\frac{1}{12}\pi^{3/2}$

(f) $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}} dx$ Ans. 3/2

(g) $\int_0^{\pi/4} \tan^4 x dx$ Ans. $\frac{\pi}{4}-\frac{2}{3}$

(h) $\int_0^{\pi^2/8} \sqrt{x}\cos^2 x^{3/2} dx$ Ans. $\frac{\pi}{3}$

(i) $\int_{-1}^{1} e^{|x|} dx$ Ans. 2e-2

(j) $\int_{1/e}^{e} |\log x| dx$ Ans. $\frac{2}{e}$

(k) $\int_0^{\pi} \frac{x}{1-\cos\alpha\sin x} dx$ Ans. $\frac{\pi(\pi-\alpha)}{\sin\alpha}$

Questions for self evaluation

1. Evaluate $\int \frac{(2x-3)dx}{x^2-3x-18}$

2. Evaluate $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$

3. Evaluate $\int \cos^4 x.dx$

4. Evaluate $\int \frac{dx}{3+2\sin x+\cos x}$

5. Evaluate $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x} dx$

6. Evaluate $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}} dx$

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$

8. Evaluate $\int_{-1}^{3/2} |x \sin \pi x| \, dx$

9. Evaluate $\int_{0}^{\pi/2} \log \sin x \, dx$

10. Evaluate $\int_{1}^{4} \left(|x-1| + |x-2| + |x-3| \right) dx$

TOPIC 7
APPLICATIONS OF INTEGRATION
SCHEMATIC DIAGRAM

Sl. No.	Topic	Concepts	Degree of Importance	Reference
7	**Applications of Integration**	Area Under the Curve y=f(x) and the lines x=a and x=b ,then area= $\int_a^b y dx$	*	NCERT Text Book Edition 2007 Ex.8.1{Q.1,2}
		Area under the curve x=f(y) and the lines y=c, y=d then area = $\int_c^d x dy$	*	Ex.8.1 (Q.3}
		Area of the region enclosed between two curves y =f(x) and y=g(x) and the lines x-a.and x=b then Area=$\int_a^b (f(x) - g(x))dx$	**	Page364,368 eg4,7 Ex 8.1Q 10 ,Ex8.2 (Q1 ,2,3},Misc Ex Q(13 ,14}
		If f(x).$\geq$ g(x) in [a,c] andf(x)$\leq$ g(x) in [c,b] a,< c <b then area =$\int_a^c (f(x) - g(x))dx$ +$\int_c^b (f(x) - g(x))dx$	***	Page370,373,374 e.g. 9,13,14 Ex8.2(Q4,5}Misc exQ 13,14
		Rough sketch of the shaded portion of the curve	*	
		Points of the Intersection of curves	*	
		Area of the regions bounded by		
		(1) Curve (Parabola, Ellipse, Circle) and a line	*	e.g.3,4,14 ,Misc Ex (Q1,2,3,6,7,8,9,10}
		Two Circles	**	e.g.10,Ex8.2 Q2
		Circle and Parabola	**	e.g.7,Ex8.2Q1, Misc Ex Q15
		Vertices of a Triangle	***	e.g.9 ,Ex8.2 Q1,Misc Ex Q13
		Sides of a Triangle	***	Ex.8.2(Q5},Misc Ex Q14
		Two Parabolas	**	e.g.6,13

ASSIGNMENTS
LEVEL I

1 Compute the area under the curve y= x^2 above x-axis and the lines x = 2 and x = 3
2. Find the area of the region bounded by the lines y-1 = x, x-axis and x = -2,x = 3.
3.Find the area enclosed by the Parabola y^2 = x and the lines y + x = 2 and x-axis 4.
4. Find the area bounded by the curve y^2=4ax and the lines y = 2a and y-axis

LEVEL II

1.Find the smaller of the two areas in which circle $x^2 + y^2 = 2a^2$ is divided by the Parabola $y^2 = ax$, $a \geq 0$

2. Find the area of the region bounded by $y = x^2 + 1$, $y = x$, $y = 2$ and $x = 0$

LEVEL III

1.Find the area of region,$\{(x, y)/ x^2+y^2 \leq 2ax, y^2 \geq ax, x, y \geq 0$

2.Find the area of the region $\{(x, y,):y^2 \leq 4x, 4x^2+4y^2 \leq 9\}$

3.Find the area bounded by the curves $y^2=4a^2(x-1)$ and the lines $x =1$ and $y = 4a$

Questions for self evaluation

1. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y – 2$.

2. Find the area bounded by the parabola $y = x^2$ and $y = |x|$.

3. Find the area of the region : $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$

4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

5. Find the area of the region : $\{(x, y): x^2 + y^2 \leq 1, \leq x + y\}$

6. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

7. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

8. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).

9. Using integration compute the area of the region bounded by the lines $2x + y = 4$, $3x – 2y = 6$, and $x – 3y + 5 = 0$.

10. Sketch the graph of : $f(x) = \begin{cases} |x-2| + 2, & x \leq 2 \\ x^2 - 2, & x > 2 \end{cases}$.

Evaluate $\int_0^4 f(x)dx$. What does the value of this integral represent on the graph ?

TOPIC
DIFFERENTIAL EQUATION
SCHEMATIC DIAGRAM

S.No	**Topic**	Concept	Degree of Importance	Reference NCERT text book class XII Maths edition 2007
8	**Differential Equation**	Order and degree of a differential equation	***	Q. 3,5,6 pg 382
		General and particular solutions of a differential equation	**	Ex. 2,3 pg384
		Formation of differential equation whose general solution is given	*	Q. 7,8,10 pg 391
		Solution of differential equation by the method of separation of variables	*	Q.4,6,10 pg 396
		Homogeneous differential equation of first order and first degree	**	Q. 3,6,12 pg 406

		Solution of differential equation of the type dy/dx +py=q where p and q are functions of x And solution of differential equation of the type dx/dy+px=q where p and q are functions of y	***	Q.4,5,10,14 pg 413,414

Assignment*s*

Level – I

1 Write the order and degree of the following differential equations

(i) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$ Ans(i) order 2 degree 2

2.Show that $y = e^{-x} + ax + b$ is the solution of $e^x \frac{d^2y}{dx^2} = 1$

3 Find the integrating factor of the differential $x\frac{dy}{dx} - y = 2x^2$ Ans 1/x

4 Solve $\frac{dy}{dx} = 1 + x + y + xy$ Ans $\log|1+y| = x + \frac{1}{2}x^2 + c$

5 Solve $(x^2 + xy)dy = (x^2 + y^2)dx$ Ans $\log|x| - \log|x - y| - \frac{y}{x} + c = 0$

6 Solve $(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ Ans $\frac{y}{x+1} = \frac{1}{3}e^{3x} + c$

7 Solve $x\frac{dy}{dx} + y = x\log x$ Ans xy = $\frac{x^2}{4}$ (2logx -1)+c

Level - II

1. Obtain the differential equation by eliminating a and b from the equation

   y = e^x(acosx + bsinx) Ans $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

2 Solve the following differential equation $(1+x^2)\frac{dy}{dx} - x = \tan^{-1}x$

Ans. y = $\frac{1}{2}\log|1+x^2| + (\tan^{-1}x)^2 + c$

3 Solve $\frac{dy}{dx} = e^{-y}\cos x$ given that y(0)=0. Ans $e^y = \sin x + 1$

4. Solve $\frac{dy}{dx} = \cos(x+y)$ Ans $\tan\left(\frac{x+y}{2}\right) = x + c$

5 Solve $\frac{dy}{dx} + 2y \tan x = \sin x$ Ans y=cosx+c cos2x

Level – III

1 Find the differential equation of the family of circles $(x - a)^2 - (y - b)^2 = r^2$

Ans $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$

2 Obtain the differential equation representing the family of parabola having vertex at the origin and axis along the positive direction of x-axis

Ans $y^2 - 2xy\frac{dy}{dx} = 0$

3 Solve $(x + y + 1)\frac{dy}{dx} = 1$ Ans. $y - \log|x + y + 2| = c$

4 Solve $xdy - ydx = \sqrt{x^2 - y^2}\,dx$ Ans. $\sin^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

5. Solve $x^2ydx - (x^3 + y^3)dy = 0$ Ans. $y = ce^{x3/3y3}$

6. Solve $ye^y dx = (y^3 + 2xe^y)dy$ Ans. $x = -y^2e^{-y} + cy^2$

7. Solve $x^2\frac{dy}{dx} = y(x + y)$ Ans $-\frac{x}{y} = \log|x| + c$

8. Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = -\frac{1}{(x^2+1)^3}$ Ans $(x^2+1)^2 = -\tan^{-1}x + c$

9. Solve the differential equation $(x + 2y^2)\frac{dy}{dx} = y$;given that when x=2,y=1

Ans $x = 2y^2$

Questions for self evaluation

1. Write the order and degree of the differential equation $\left(\frac{d^3y}{dy^3}\right)^2 + \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$

2. Form the differential equation representing the family of ellipses having foci on x –axis and centre at origin .

3. Solve the differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$, given that y = 0 when x = 0.

4. Solve the differential equation : $xdy - y\,dx = \sqrt{x^2 + y^2}\,dx$

5. Solve the differential equation : $x \log x \frac{dy}{dx} + y = \frac{2}{x}\log x$.

6. Solve the differential equation : $x^2 dy + (y^2 + xy)\,dx.= 0$, $y(1) = 1$.

7. Show that the differential equation $2y.e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous and find its particular solution given that $y(0) = 1$.

8. Find the particular solution of differential equation

$$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x, \text{ given that } y\left(\frac{\pi}{2}\right) = 0 .$$

TOPIC 9
VECTOR ALGEBRA
SCHEMATIC DIAGRAM

Sr no.	Topic	Concept	Degree of importance	Refrence NCERT Text Book Edition 2007
9.	Vector algebra			
		Vector and scalars	*	Q2 pg428
		Direction ratio and direction cosines	*	Q 12,13 pg 440
		Unit vector	* *	Ex 6,8 Pg 436
		Position vector of a point and collinear vectors	* *	Q 15 Pg 440 Q 11Pg440 Q 16 Pg448
		Dot product of two vectors	**	Q6 ,13 Pg445
		Projection of a vector	* * *	Ex 16 Pg 445
		Cross product of two vectors	* *	Q 12 Pg458
		Area of a triangle	*	Q 9 Pg 454
		Area of a parallelogram	*	Q 10 Pg 455

Level 1

1 mark questions

1. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 9\hat{k}$ find a unit vector parallel to $\vec{a} + \vec{b}$.

Ans: $\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

2. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$

Ans: $5\hat{i} - 10\hat{j} + 10\hat{k}$

3 Find the position vector of the mid point of the line segment joining the points A($5\hat{i} + 3\hat{j}$) and B($3\hat{i} - \hat{j}$).

Ans. $4\hat{i} + \hat{j}$

4. In a triangle ABC, the sides AB and BC are represents by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

Ans: $\overrightarrow{CA} = -(3\hat{i} + 2\hat{j} + 7\hat{k})$

5 For what values of λ, vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{b} = \lambda\hat{i} - 4\hat{j} + 8\hat{k}$ are

Orthogonal (ii) Parallel ` Ans: (i) $\lambda = \frac{-40}{3}$ (ii) $\lambda = 6$

6. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$. Ans: $|\vec{x}| = 4$

7. Find $\vec{a} . \vec{b}$ if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$. Ans: $\vec{a} . \vec{b} = 9$

8 If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} . \vec{b} = \sqrt{6}$. Then find the angle between $\vec{a}$ and $\vec{b}$. Ans: $\frac{\pi}{4}$

9 Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} . \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$. Ans: $\frac{8}{7}$

10 If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a Parallelogram, then write the area of Parallelogram in terms of $\vec{a}$ and $\vec{b}$. Ans: $\theta = \frac{\pi}{4}$

11 Find the area of Parallelogram whose adjacent sides are represented by the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$. Ans: $10\sqrt{3}$ Sq. units

4 marks questions

1. Show that the area of the Parallelogram having diagonals $(3\hat{i} + \hat{j} - 2\hat{k})$ and $(\hat{i} - 3\hat{j} + 4\hat{k})$ is $5\sqrt{3}$ Sq units.

2. Find the area of triangle having the points A(1,1,1) , B(1,2,3) and C(2,3,1) as its vertices.

Ans: $\frac{\sqrt{21}}{2}$ Sq units

Level II

1mark questions

1 If a line make α, β, γ with the X - axis , Y– axis and Z – axis respectively, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ Ans: 2

2 For what value of p, is $(\hat{i} + \hat{j} + \hat{k})\,p$ a unit vector? Ans: $P = \pm\frac{1}{\sqrt{3}}$

3 What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis

Ans: Cosine of the angle with y –axis is $\frac{1}{2}$

4 Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors. Ans: $P = \frac{2}{3}$

5 Find the value of the following: $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$ Ans: 1

6 Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.

Ans: $\theta = \frac{\pi}{6}$

7 Find λ, if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$. Ans: $\lambda = -3$

8 Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$ Ans: $\theta = \frac{\pi}{3}$

9 If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} . \vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$. Ans: $\theta = \frac{\pi}{4}$

10 If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} . \vec{b} = 9$. Find $|\vec{a} \times \vec{b}|$ Ans: 12

4 marks questions

1. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

2. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a} + \vec{b})$, $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{j} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$. Ans: $5(\frac{-1}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k})$

3. Write the position vector of a point R which divides the line joining the points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2 : 1 externally.

Ans: $-3\hat{i} + 3\hat{k}$

4. The dot products of a vector with the vectors $\hat{i} - 3\hat{k}$, $\hat{i} - 2\hat{k}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. Find the vectors. Ans: $\vec{a} = 15\hat{i} - 27\hat{j} + 5\hat{k}$

5 If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, find the value of λ. Ans: $\lambda = 8$

6 Three vertices of a triangle are A(0, -1, -2), B(3,1,4) and C(5,7,1). Show that it is a right angled triangle. Also find the other two angles. Ans: 45^0, 45^0

7 . If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$.

8 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} . \vec{c} = 3$. Ans: $\vec{c} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

Level III
1mark questions

1 If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$. Ans: $\frac{\pi}{2}$

4 marks questions

1 Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

2 The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ. Ans: $\lambda=\frac{1}{2}$

3 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$. Ans: $5\sqrt{2}$

4. If a unit vector $\vec{a}$ makes angles $\pi/4$, with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component of $\vec{a}$ and angle θ. $[\frac{1}{\sqrt{2}}\hat{i}, \frac{1}{2}\hat{j}, \frac{1}{2}\hat{k},\ \theta = \pi/3]$

5 If $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ show that $(\vec{a}-\vec{d})$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

6 . If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices of a Δ ABC, show that the area of the Δ ABC is $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$.

7. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{a}\times\vec{b})$.

8 . Let $\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$, $\vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 18$. Ans: $\vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$

9. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, $\vec{\alpha} = 3\hat{i}-\hat{j}$, $\vec{\beta} = 2\hat{i}+\hat{j}-3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. Ans: $\vec{\beta}_1 = \frac{1}{2}(3\hat{i}-\hat{j})$, $\vec{\beta} = \frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

Questions for self evaluation

1. Show that the points A, B, C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non - zero and non – coplanar vectors, prove that $\vec{a} - 2\vec{b} + 3\vec{c}$, $\vec{a} - 3\vec{b} + 5\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are also coplanar.

3. Dot product of a vector with $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$, and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5, 8 respectively. Find the vector.

4. Find the components of a vector which is perpendicular to the vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

5. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

6. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

7. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that

(i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (ii) $\tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$

8. Express $2\hat{i} - \hat{j} + 3\hat{k}$ as the sum of a vector parellal and perpendicular to $2\hat{i} + 4\hat{j} - 2\hat{k}$.

TOPIC 10
THREE DIMENSIONAL GEOMETRY
SCHEMATIC DIAGRAM

Sr no.	Topic	Concept	Degree of importance	Refrence NCERT Text Book Edition 2007
10	Three Dimensional Geometry	Direction Ratios and Direction Cosines	*	Ex No 2 Pg -466 Ex No 5 Pg – 467 Ex No 14 Pg - 480
		Cartesian and Vector equation of a line in space & conversion of one into another form	**	Ex No 8 Pg -470 Q N. 6, 7, - Pg 477 QN 9 – Pg 478
		Co-planer and skew lines	*	Ex No 29 Pg -496
		Shortest distance between two lines	***	Ex No 12 Pg -476 Q N. 16, 17 - Pg 478
		Cartesian and Vector equation of a plane in space & conversion of one into another form	**	Ex No 17 Pg -482 Ex No 18 Pg – 484 Ex No 19 Pg – 485 Ex No 27 Pg – 495 Q N. 19, 20 - Pg 499
		Angle Between (iv) Two lines (v) Two planes (vi) Line & plane	 * * **	Ex No 9 Pg -472 Q N. 11 - Pg 478 Ex No 26 Pg – 494 Q N. 12 - Pg 494 Ex No 25 Pg - 492
		Distance of a point from a plane	**	Q No 18 Pg -499 Q No 14 Pg – 494
		Distance measures parallel to plane and parallel to line	**	
		Equation of a plane through the intersection of two planes	***	Q No 10 Pg -493
		Foot of perpendicular and image with respect to a line and plane	**	Ex. N 16 Pg 481

Level-I
1 mark questions

1. Find the distance of the point (a,b,c) from x-axis [Ans. $\sqrt{b^2+c^2}$]
2. Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point(1,2,3). $\left[\text{Ans. } \frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{6}\right]$

3.Find the direction cosines of the line passing through the following points (-2,4,-5), (1,2,3).

$$\left[\text{Ans.}\frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right]$$

4.Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$. Ans.$\vec{r} = (5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k}$

5.Write the distance of plane 2x– y + 2z + 1 = 0 from the origins.

Ans. 1/3

6.Express the equation of the line $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ in the Cartesian form.

Ans. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-1}{2} = \lambda$

7. Express the equation of the plane $\vec{r} \cdot (2\hat{\imath} - 3\hat{\jmath} + \hat{k}) + 4 = 0$ in the Cartesian form .

Ans. 2x – 3y + z + 4 = 0

8. Find the equation of plane with intercepts 2, 3, 4 on the x ,y, z –axis respectively.

Ans. 12x + 4y + 3z = 12

9.Find the value of p, such that the lines $\frac{x}{1} = \frac{y}{3} = \frac{z}{2p}$ and $\frac{x}{-3} = \frac{y}{5} = \frac{z}{2}$ are perpendicular to each other.

Ans. p = –3

10. Find the angle between the lines whose direction ratios are (1, 1, 2) and ($\sqrt{3}$–1 , –$\sqrt{3}$ –1 , 4).

Ans. 60^0

4 marks questions

1. Find the shortest distance between the lines l_1 and l_2 given by the following:

(a) $l_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ $l_2 : \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$ Ans.$\frac{3\sqrt{2}}{2}$ units ,

(b) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$

$\vec{r} = (4\hat{\imath} + 2\mu)\,\hat{\imath} + (5 + 3\mu)\,\hat{\jmath} + (6 + \mu)\,\hat{k}$. Ans $\frac{3}{\sqrt{19}}$ units

2.Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

Ans.(4 , 3, 7) &(-2,-1,3)

6 marks questions

1.Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

2.Find the equation of the plane passing through the point (1,1,1) and containing the line

$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$. Also, show that the plane contains the line

$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} - 5\hat{k})$. Ans. $x - 2y + z = 0$

LEVEL II

1 mark questions

1.Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane

$3x - y - 2z = 7$. Ans. $\lambda = -3$

2. Write the direction cosines of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$. Ans. $-\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}$

3. Find the equation of a plane passing through the origin and perpendicular to x-axis.

Ans. $x = 0$

4. Find the angle between line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane $3x + 4y + z + 5 = 0$.

Ans. $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$

5.Find the distance between the parallel planes $x + y - z = -4$ and $2x + 2y - 2z + 10 = 0$.

Ans. $\frac{1}{\sqrt{3}}$

4marks questions

1. Find whether the lines $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not.

   If intersecting , find their point of intersection.

   Ans. Lines are intersecting & point of intersection is (3,0,-1).

2. Show that the four points (0,-1,-1) , (4,5,1) , (3,9,4) and (-4,4,4,) are coplanar. Also, find

   the equation of the plane containing them. Ans. $5x - 7y + 11z + 4 = 0$

3.Find the equation of plane passing through the point (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5). Also find the perpendicular distance of the plane from the origin.

Ans. $x - y + 3z - 2 = 0$, $\frac{2\sqrt{11}}{11}$

4. Find the Cartesian equation of the plane passing through the points A(0,0,0) and

   B(3,-1,2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ ` Ans. $x - 19y - 11z = 0$

5. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and

   which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

Ans. $51x + 15y - 50z + 173 = 0$

6. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3,0,-4). Also, find the distance between these two lines.

Ans. Vector equation $\vec{r} = (3\hat{\imath} - 4\hat{k}) + \lambda(5\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$ and distance = 7.75 units

6marks questions

1.The points A(4,5,10) , B(2,3,4) and C(1,2,-1) are three vertices of a parallelogram ABCD. Find the vector equation of the sides AB and BC and also find the coordinates

Ans. Equation of AB is $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 10\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$.

Equation of BC is $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + 5\hat{k})$. Coordinates of D are (3,4,5).

2Find the equations of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point (1,1,1). Ans. $\frac{x-1}{-2} = \frac{y-1}{10} = \frac{z-1}{17}$

LEVEL III

1 Mark questions

1. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axis. Ans. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

2. Find the point through which the line $2x = 3y = 4z$ passes. Ans. (0, 0, 0)

4 Marks questions

1.Find the equation of the perpendicular drawn from the point P(2,4,-1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Ans. $\frac{x-2}{-6} = \frac{y-4}{-3} = \frac{z+1}{-2}$

2. Find the equation of the plane that contains the point (1,-1,2) and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$. Ans. $5x - 4y - z = 7$

3. Find the distance of the point P(6,5,9) from the plane determined by the points A(3,-1,2) , B(5,2,4) and C(-1,-1,6). Ans. $\frac{6}{\sqrt{34}}$

4. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

6 Marks questions

1. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane 2x – y + z + 3 = 0. Find also, the image of the point in the plane.

Ans. Foot of perpendicular (-1,4,3), Image (-3,5,2), Distance $= \sqrt{6}$ units

2. Find the coordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by points A(1,2,3) , B(2,2,1) and C(-1,3,6). Ans :(1, -2, 7)

3. Find the foot of the perpendicular from P(1,2,3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, obtain the equation of the plane containing the line and the point (1,2,3). Ans : (3, 5, 9)

4. Find the equation of the plane containing the lines $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Find the distance of this plane from origin and also from the point (1,1,1).

Ans. $x + y - z - 2 = 0$, $\frac{2}{\sqrt{3}}$ *units* , $\frac{1}{\sqrt{3}}$ units.

5.Prove that the image of the point (3,-2,1) in the plane 3x – y + 4z =2 lies on the plane,

x + y + z + 4 = 0. Ans. Image of the point = (0,-1,-3)

6. Find the distance of the point (3,4,5) from the plane x + y + z =2 measured parallel to the

line 2x = y = z. Ans. 6 units

Questions for self evaluation

1. Find the vector equation of a line joining the points with position vectors $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$, and $2\hat{i} + \hat{j} + 2\hat{k}$. Also find the Cartesian equivalent of this equation.
2. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.
3. Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).
4. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}), \text{ and } \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$$

5. Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.
6. Show that The four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .
7. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane.
8. Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

TOPIC 11
LINEAR PROGRAMMING
SCHEMATIC DIAGRAM

S.No	Topic	Concepts	Degree of Importance	References From NCERT Book Vol. II
11	Linear Programming	(i) LPP and its Mathematical Formulation	**	**Articles 12.2 and 12.2.1**
		(ii) Graphical method of solving LPP (bounded and unbounded solutions)	**	**Article 12.2.2** Solved Examples 1 to 5 Q. Nos 5 to 8 Ex.12.1
		(iii) Types of problems (a) Diet Problem	***	Q. Nos 1, 2 and 9 Ex. 12.2 Solved Example 9 Q. Nos 2 Misc. Ex.
		(b) Manufacturing Problem	***	Solved Example 8 Q. Nos 3 of Ex. 12.2 Solved Example10
		(c) Allocation Problem	**	Solved Example 7 Q. No. 10 Ex.12.2, Q. No. 5 Misc. Ex.
		(d) Transportation Problem	*	Solved Example11 Q. No. 7 Misc. Ex.
		(e) Miscellaneous Problems	**	Q. No. 8 Ex. 12.2

Assignments on Concept (iii)

LEVEL I

1. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of mineral and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost? Also find the least cost.

[**Answer :** Least cost = Rs.110 at $x = 5$ and $y = 30$.]

2. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 10 per kg and rice Rs. 20 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 gm and 200 gm respectively. In what quantities, should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost ?

[**Answer :** Minimum cost = Rs.8 at $x = 400$ and $y = 200$

Or x=800 and y = 0]

3. A company manufactures two articles A and B. There are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs. 6 for each unit of A and Rs. 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

[**Answer :** Maximum profit is Rs. 120 when 12 units of A and 6 units of B are produced.]

4. A company sells two different produces A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125. Profit on each unit of A is Rs. 20 and that on B is Rs. 15. How many units of A and B should be produced to maximize the profit? Solve it graphically. [**Answer :** For maximum profit, 25 units of product A and 125 units of product B are produced and sold.]

5. A company manufactures, two types of toys-A and B. Toy A requires 4 minutes for cutting and 8 minutes for assembling and Toy B requires 8 minutes for assembling. There are 3 hours and 20 minutes available in a day for cutting and 4 hours for assembling. The profit on a piece of toy A is Rs. 50 and that on toy B is Rs. 60. How many toys of each type should be made daily to have maximum profit? Solve the problem graphically.

[**Answer :** For Maximum profit, no toy of type A and 30 toys of type B should be manufactured.]

6. A manufacture makes two types of cups, A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below:

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise, and on B it is 50 paise, show that the 15 cups of type A and 30 cups of type B should be manufactured per day to get the maximum profit.

LEVEL II

7. Ramesh wants to invest at most Rs. 70,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 10,000 in Bond A and at least Rs. 30,000 in Bond B. If the rate of interest on bond A is 8 % per annum and the rate of interest on bond B is 10 % per annum , how much money should he invest to earn maximum yearly income ? Find also his maximum yearly income.

[**Answer :** Maximum annual income = Rs. 6,800 on investment of Rs. 10,000 on Bond A and Rs. 60,000 on Bond B.]

8. A factory owner wants to purchase two types of machines, A and B, for his factory, The machine A requires an area of 1000 m^2 and 12 skilled men for running it and its daily output is 50 units whereas the machine B requires an area of 1200 m^2 and 8 skilled men, and its daily output is 40 units. If an area of 7600 m^2 and 72 skilled men be available to operate the machine, how many machines of each should be bought to maximize the daily output?

[**Answer :** Output is maximum when 4 machine A and 3 machine B should be purchased.]

9. A shopkeeper deals in two items- thermos flasks and air tight containers. A flask costs him Rs. 120 and an air tight container costs him Rs. 60. He has at the most Rs. 12,000 to invest and has space to store a maximum of 150 items. The profit on selling a flask is Rs. 20 and an air tight container is Rs. 15. Assuming that he will be able to sell all things he buys, how many of each items should he buy to maximize his profit? Solve the problem graphically.

[**Answer :** For maximum profit, 50 flasks and 100 containers should be bought and sold.]

10. An oil company requires 12,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimize the cost of requirement?

[**Answer :** A should run for 60 days and B for 30 days.]

11. A furniture dealer deals only in two items — tables and chairs. He has Rs. 10,000 to invest and a space to store at most 60 pieces. A table costs him Rs. 500 and a chair Rs. 200. He can sell a table at a profit of Rs. 50 and a chair at a profit of Rs. 15. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

[**Answer :** Maximum profit = Rs. 1000 when no chair is sold and only 20 tables are sold.]

12. A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% of nitrogen and 10% of phosphoric acid. After soil testing it is found that at least 14 kg of nitrogen and the same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs Rs. 2.00 per kg and the type B costs Rs. 3.00 per kg. Using linear programming find how many kgs of each type of the fertilizer should be bought to meet the requirement and the cost be minimum. Solve the problem graphically.

[**Answer :** Cost is minimum when 100 kg of type A and 80 kg of type B fertilizers are used.]

LEVEL III

13. If a young man rides his motorcycle at 25km/hour, he had to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/hour, the petrol cost increases at Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.

[**Answer :** Maximum distance = 30 km, where x = 50/3 and y = 40/3}

14. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to to stitch at least 60 shirts and 32 pants at a minimum labour cost.

[**Answer :** Cost is minimum when tailor A and tailor B work for 5 days and 3 days respectively.]

15. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each executive class ticket and a profit of Rs 350 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

[**Answer :** For maximum profit, 62 executive class tickets and 188 economy class ticket should be sold.]

16.A medicine company has factories at two places A and B . From these places, supply is to be made to each of its three agencies P, Q and R. The monthly requirement of these agencies are respectively 40, 40 and 50 packets of the medicines, While the production capacity of the factories at A and B are 60 and 70 packets are respectively. The transportation cost per packet from these factories to the agencies are given:

Transportation cost per packet (in Rs.)		
From / To	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost.

[**Answer :** Minimum transportation cost is Rs. 400 when 10, 0 and 50 packets are transported from factory at A and 30, 40 and 0 packets are transported from factory at B to the agencies at P, Q and R respectively.]

Questions for self evaluation

1. Solve the following linear programming problem graphically : Maximize $z = x - 7y + 190$ subject to the constraints $x + y \leq 8$, $x \leq 5$, $y \leq 5$, $x + y \geq 4$, $x \geq 0$, $y \geq 0$.

2. Solve the following linear programming problem graphically : Minimize $z = 3x + 5y$ subject to the constraints $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$.

3. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains, 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs. 5 per kilogram and rice costs Rs. 4 per kilogram.

4. A shopkeeper deals only in two items — tables and chairs. He has Rs. 6,000 to invest and a space to store at most 20 pieces. A table costs him Rs. 400 and a chair Rs. 250. He can sell a table at a profit of Rs. 25 and a chair at a profit of Rs. 40. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

5. A small firm manufactures items A and B. The total number of items A and B it can manufacture a day is at most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs. 300 and one unit of item B be Rs. 160, how many of each type of item be produced to maximize the profit ? Solve the problem graphically.

6. A chemist requires 10, 12 and 12 units of chemicals A, B and C respectively for his analysis. A liquid product contains 5, 2, and 1 units of A, B and C respectively and it costs Rs. 3 per jar. A dry product contains 1, 2, and 4 units of A, B and C per carton and costs Rs. 2 per carton. How many of each should he purchase in order to minimize the cost and meet the requirement ?

7. A person wants to invest at most Rs. 18,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 4,000 in Bond A and at least Rs. 5,000 in Bond B. If the rate of interest on bond A is 9 % per annum and the rate of interest on bond B is 11 % per annum , how much money should he invest to earn maximum yearly income ?

8. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to to stitch at least 60 shirts and 32 pants at a minimum labour cost.

TOPIC 12
PROBABILITY
SCHEMATIC DIAGRAM

S.No	Topic	Concepts	Degree of Importance	References From NCERT Book Vol. II
12	Probability	(i) Conditional Probability	***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 ,15 Ex. 13.1
		(ii)Multiplication theorem on probability	**	**Article 13.3** Solved Examples 8 & 9 Q. Nos 2, 3, 13 14 & 16 Ex.13.2
		(iii) Independent Events	***	**Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2
		(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 Q. Nos 4,5,10,12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
		(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1,6,8&15Ex. 13.4
		(vi) Bernoulli,s trials and Binomial Distribution	***	**Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 Ex.13.5

Assignments on Concepts (i) , (ii) and (iii)

LEVEL I

1. 1. A coin is tossed thrice and all 8 outcomes are equally likely.
   E : "The first throw results in head"
   F : "The last throw results in tail"
   Are the events independent ? [**Answer :** Yes**]**

2. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{4}$. Are the events independent ? [**Answer :** Yes**]**

3. If A and B are independent events, Find P(B) if $P(A \cup B) = 0.60$ and $P(A) = 0.35$. [**Answer :** $\frac{5}{13}$]

4. If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive events. [**Answer : 0]**

5. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced. [**Answer :** $\frac{5}{14}$]

LEVEL II

1. A dice is thrown twice and sum of numbers appearing is observed to be 6. what is the conditional probability that the number 4 has appeared at least once. [**Answer :** $\frac{2}{5}$]

2. A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what is the probability that none is red. [**Answer :** $\frac{8}{65}$]

3. The probability of A hitting a target is $\frac{3}{7}$ and that of B hitting is $\frac{1}{3}$. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.

[**Answer (i)** $\frac{13}{21}$ (ii) $\frac{10}{21}$]

LEVEL III

1. A class consists of 80 students; 25 of them are girls and 55 are boys, 10 of them are rich and the remaining poor; 20 of them are fair complexioned. what is the probability of selecting a fair complexioned rich girl. [**Answer :** $\frac{5}{512}$ **]**

2. If P(A) = $\frac{5}{8}$, P(B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{2}$, find $P(\overline{A}/\overline{B})$ and $P(\overline{B}/\overline{A})$ [**Answer :** $\frac{3}{4}$ *and* $\frac{3}{4}$ **]**

3. Two integers are selected from integers 1 through 11. If the sum is even, find the probability that both the numbers are odd. [**Answer :** $\frac{3}{5}$]

Assignment on Concept (iv)

LEVEL I

1. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag . Find the probability that the ball drawn is blue in colour. [**Answer :**93/154]

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts . Find the probability of the lost card being a heart.

[**Answer :** $\frac{11}{50}$]

3. An insurance company insured 2000 scooter and 3000 motorcycles . The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02 . An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle. [**Answer :** $\frac{3}{4}$]

4. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin.

[**Answer :** 19/42]

5. Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

[**Answer :** 0.27]

LEVEL II

1. Find the probability of drawing a one-rupee coin from a purse with two compartments one of which contains 3 fifty-paise coins and 2 one-rupee coins and other contains 2 fifty-paise coins and 3 one-rupee coins. [**Answer :** 1/2]

2. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women. [**Answer :** 2/3]

3. A company has two plants to manufacture bicycles. The first plant manufactures 60 % of the bicycles and the second plant 40 % . Out of that 80 % of the bicycles are rated of standard quality at the first plant and 90 % of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.
[**Answer :** 3/7]

LEVEL III

1. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from
(i) LONDON
(ii) CLIFTON ? [**Answer :** (i) 12/17 (ii) 5/17]

2. A test detection of a particular disease is not fool proof. The test will correctly detect the disease 90 % of the time, but will incorrectly detect the disease 1 % of the time. For a large population of which an estimated 0.2 % have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease.
[**Answer :** 0.15]

Assignments on Concepts (v) and (vi)

LEVEL I

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of spades.

Ans

X	0	1	2
P(X)	9/16	6/16	1/16

2. 4 defective apples are accidentally mixed with 16 good ones. Three apples are drawn at random from the mixed lot. Find the probability distribution of the number of defective apples.

Ans.

X	0	1	2	3
P(X)	28/57	24/57	24/285	1/285

3. A random variable X is specified by the following distribution

X	2	3	4
P(X)	0.3	0.4	0.3

Find the variance of the distribution. [**Answer :** 0.6]

LEVEL II

1. If a die is thrown 5 times, what is the chance that an even number will come up exactly 3 times.
[**Answer :** 5/16]

2. An experiment succeeds twice as often it fails. Find the probability that in the next six trials, there will be at least 4 success. [**Answer :** 496/729]

3. A pair of dice is thrown 200 times. If getting a sum 9 is considered a success, find the mean and variance of the number of success.

$$\left[\text{Ans. } \frac{200}{9}, \frac{1600}{81}\right]$$

LEVEL III

1. A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice. Find the probability distribution of the number of tails.

Ans.

X	0	1	2
P(X)	9/16	6/16	1/16

2. The sum of mean and variance of a binomial distribution for 5 trials be 1.8. Find the probability distribution. $\left[\text{Ans.} \left(\frac{4}{5}+\frac{1}{5}\right)^5\right]$

3. The mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively. Find $P(X \geq 1)$.

$$\left[\text{Ans. } \frac{65}{81}\right]$$

Questions for self evaluation

1. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5. 1/4

2. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of an experiment are performed. Find the probability that the event happens at least once. 0.784

3. A football match is either won, draw or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches. 1/9.

4. A candidate has to reach the examination center in time. Probability of him going by bus ore scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively. But he reaches in time if he uses other mode of transport. He reached late at the centre. Find the probability that he traveled by bus. 9/13

5. Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0, \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \end{cases} \quad \text{, k is a + ve constant.}$$

Find the mean and variance of the probability distribution. 19/8, 47/64

6. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.625/23328

7. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ? 11/243

8. Two cards are drawn simultaneously (or successively) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards. 1, and 1.47

ANSWERS OF QUESTIONS FOR SELF EVALUATION

1.Relations & Functions

2. {1, 5, 9}   3. T_1 is related to T_3 6. $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$

7. e = 0, $a^{-1} = \frac{a}{a-1}$ 8. Identity element (1 , 0), Inverse of (a , b) is $\left(\frac{1}{a}, \frac{-b}{a}\right)$

2.Inverse Trigonometric Functions

6. x   7. $\pm\frac{1}{\sqrt{2}}$ 8. $\frac{1}{6}$

3.Matrices and determinants

4. $\begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & 1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$ 7. x = 8, y = 8 and $A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

8. $A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$, x = 2, y = 1, z = 3   9. $AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, x = 3, y = – 2, z = –1

10. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

4. Continuity& Differentiability

1. k = 5   2. a = 3, b = 2,   4. $a = -\frac{3}{2}, c = \frac{1}{2}, b \in R$

5. $\left[\frac{dy}{dx}\right]_{\theta=\pi/4} = 1$ 6. $(\log x)^{\cos x}\left[\frac{\cos x}{x \log x} - \sin x.\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$

7. $\frac{y - \sec^2 x}{1 - x - 2y}$ 10. $\frac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$

5. Application of derivatives

1. $\frac{1}{48\pi}$ cm/s   2. $b\sqrt{3}cm^2/s$ 3. ↑ in (– 2 , – 1) and ↓ in $(-\infty,-2) \cup (-1,\infty)$

4. ↑ in $\left[0,\frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ and ↓ in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 5. (0 , 0) , (1 , 2), (– 1, – 2)

6. (a) y – 2x – 3 = 0 , (b) 36y + 12x – 227 = 0   8.(i) 2.962 (ii) 2.0018

10. $\frac{200}{7}$ m

6.Indefinite & Definite Integrals

1. $\log\left|x^2 + 3x - 18\right| + c$ 2. $-3\sqrt{5 - 2x - x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

3. $\left[\frac{1}{8}\left(3x+2\sin 2x+\frac{\sin 4x}{4}\right)+c\right]$

4. $\tan^{-1}\left(1+\tan\frac{x}{2}\right)+c$

5. $\frac{18}{15}x+\frac{1}{25}\log|3\sin x+4\cos x|+c$

6. $x-\sqrt{1-x^2}\sin^{-1}x+c$

7. $\frac{64}{231}$

8. $\frac{3}{\pi}+\frac{1}{\pi^2}$

9. $-\frac{\pi}{2}\log 2$

10. 19/2

7.Applications of Integration

1. $\frac{9}{8}$ sq. units

2. $\frac{1}{3}$ sq. units

3. $\frac{23}{6}$ sq. units

4. $\frac{3}{2}(\pi-2)$ sq. units

5. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq. units

6. $\frac{4}{3}(8+3\pi)$ sq. units

7. $\left(\frac{2\pi}{3}-2\sqrt{3}+4\sin^{-1}\frac{1}{4}\right)$ sq. units

8. 18 sq. units

8. Differential equations

1. Order 2, Degree not defined

2. $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

3. $x=(\tan^{-1}y-1)+Ce^{-\tan^{-1}y}$

4. $y+\sqrt{x^2+y^2}=Cx^2$

5. ylogx = $\frac{-2}{x}(1+\log|x|)+C$

6. y + 2x = $3x^2y$

7. $2e^{\frac{x}{y}}+\log|y|=2$

8. $y=x^2-\frac{\pi^2}{4\sin x}$

9. Vector Algebra

3. $\hat{i}+2\hat{j}+\hat{k}$

4. $\pm\frac{3}{\sqrt{83}}\vec{i},\mp\frac{5}{\sqrt{83}}\vec{j},\mp\frac{7}{\sqrt{83}}\vec{k}$

6. $5\sqrt{2}$

8. $\left(-\frac{\hat{i}}{2}-\hat{j}+\frac{\hat{k}}{2}\right)+\frac{5}{2}(\hat{i}+\hat{j})$

10. Three Dimensional Geometry

1. $\vec{r}=(\hat{i}-2\hat{j}-3\hat{k})+\lambda(\hat{i}+2\hat{j}-2\hat{k})$, $\frac{x-1}{1}=\frac{x+2}{2}=\frac{z+3}{-2}$

2. $\lambda=-1$, $\mu=-1$, (–1, –1, –1)

3. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

4. ZERO

5. $\left(\frac{39}{7},\frac{-6}{7},\frac{-37}{7}\right)$

6. 5x – 7y + 11z + 4 = 0

7. 12x – 4y + 3z = 169

8. 17x + 2y – 7z = 12

11. Linear Programming

1. Minimum 195 at (0 , 5).

2. Minimum value is 7 at $\left(\frac{3}{2},\frac{1}{2}\right)$

3. Maximum is Rs 4.60 at (0.6 , 0.4)
4. Maximum is Rs.800 at (0 , 20)
5. 8 items of type A and 16 items of type B
6.1 jar of liquid and 5 cartons of dry product.
7. Rs.4,000 in Bond A and Rs.14,000 in Bond B
8. Minimum cost Rs.1350 at (5 , 3)

12. Probability

1. $\frac{1}{4}$ 2. 0.784 3. $\frac{1}{9}$

4. $\frac{9}{13}$ 5. $\frac{19}{8}, \frac{47}{64}$ 6. $\frac{625}{23328}$

7. $\frac{11}{243}$ 8. 1 and 0.5